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Multiple-bank lending: Diversification and free-riding in monitoring

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Abstract

This paper analyzes the optimality of multiple-bank lending, when firms and banks are subject to moral hazard and monitoring is essential. Multiple-bank lending leads to higher per-project monitoring whenever the benefit of greater diversification dominates the costs of free-riding and duplication of effort. The model predicts a greater use of multiple-bank lending when banks have lower equity, firms are less profitable and monitoring costs are high. These results are consistent with some empirical observations concerning the use of multiple-bank lending in small and medium business lending.

JEL classification: D82; G21; G32

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1. Introduction

There seems to be a wide consensus among economists on the role that banks perform in the economy. The theoretical literature portrays banks as reducing information asymmetries between investors and borrowers. In originating loans and monitoring borrowers, banks acquire private information about their customers and enhance the value of investment projects (for example, Boot

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and Thakor, 2000). The empirical literature supports this view, and suggests improved project payoffs as the special feature of bank lending relative to capital market lending (see, for example, the review in Ongena and Smith, 2000a).

Despite the emphasis on the monitoring role of banks, the issue of the optimal number of monitors remains unclear. According to the theory of financial intermediation, if banks can expand indefinitely and achieve fully diversified portfolios, they exert the first best monitoring level and have no (or low) default risk (Diamond, 1984; Ramakrishnan and Thakor, 1984; and also Allen, 1990). Thus exclusive bank–firm relationships involving a single monitor are optimal since they avoid free-riding problems and duplication of monitoring efforts. While certainly being appealing, this prediction seems at odds with the fact that in reality banks are of finite size and exclusive bank–firm relationships are often not observed. For example, Ongena and Smith (2000b) document that less than 15% of the firms in a sample from twenty European countries maintain a single relationship. Moreover, even if the number of bank relationships tends to increase with firm size, also small and medium enterprises (SMEs) borrow from more than one bank at some point in their life cycle as reported for countries like the US, Italy and Portugal (Petersen and Rajan, 1994; Detragiache et al., 2000; and Farinha and Santos, 2002).

These empirical observations raise a number of important questions. If monitoring is one of the main functions of banks, why should banks share firms' financing if this leads to a diminished role for their monitoring? Does the widespread use of multiple-bank lending suggest that the role of banks as delegated monitors is of minor importance? Or does multiple-bank lending entail some—previously unnoticed—benefits for banks' incentives to monitor? These questions are of particular importance in contexts where monitoring is essential due to information opacity and the need to process soft information, like small and medium business lending (see, for example, Cole et al., 2004).

To better understand the role of banks as monitors in the context of multiple bank relationships, we present a simple model where banks face limited diversification opportunities so that, differently from Diamond (1984) and Ramakrishnan and Thakor (1984), they cannot construct fully diversified portfolios. In this context, we show that multiple-bank lending may allow banks to mitigate the agency problem with depositors and achieve higher monitoring and expected profits.

Our starting point is a simple one-period model of bank lending with double moral hazard, where banks of limited size raise deposits from investors and grant loans to entrepreneurs. Firms need external funds to undertake investment projects and can privately decide whether to exert effort and increase project success probabilities. Banks can ameliorate firms' moral hazard problem through monitoring, which is, however, costly and not observable.

The unobservability of monitoring introduces another moral hazard problem between banks and depositors. Given that banks cannot perfectly diversify when lending individually, their incentives to monitor depend on the level of equity they have, the cost of monitoring, the profitability of firms, and most importantly, on whether they lend to firms individually or share lending with other banks. Multiple-bank lending allows the financing of more independent projects. Greater diversification improves banks' monitoring incentives, as it reduces the variance of the return of their portfolios and allows banks to be residual claimants of any additional marginal benefit of monitoring. This lowers deposit rates and improves monitoring incentives further. At the same time, however, since banks do not coordinate on their monitoring choices and project success probabilities depend on the effort of all of them, multiple-bank lending involves free-riding and duplication of effort. When the agency problem between banks and depositors is sufficiently severe, the benefit of greater diversification dominates the drawbacks of free-riding and duplication of effort, and multiple-bank lending leads to higher per-project monitoring than individual-bank lending.

The model predicts that the attractiveness of sharing lending decreases with the amount of banks' equity and firms' prior profitability, while it increases with the cost of monitoring. In line with our prediction on firms' profitability, Petersen and Rajan (1994), Detragiache et al. (2000), Farinha and Santos (2002), and Guiso and Minetti (2006) document a greater use of multiple-bank lending for firms with lower prior profitability. As for the cost of monitoring, the results in Guiso and Minetti (2006) indicate that less opaque firms for which the cost of monitoring is lower borrow more from individual lenders.

The main insight of the paper is to provide in a static model a new explanation for the use of multiple-bank lending as a way to improve monitoring incentives. The analysis is suited for the study of bank–firm relationships when monitoring is important and banks need greater diversification to improve the value of the relationships as is the case in small and medium business lending.[1] The model hinges around two features—leverage and limited diversification opportunities—that have been identified as important in banking (see, for example, Marquez, 2002, and other papers cited therein). The incentive mechanism of diversification works only if banks raise deposits. Otherwise, diversification reduces the variance of the return of banks' portfolios but has no impact on their monitoring incentives.

The previous literature on the number of bank relationships has explained multiple-bank lending in terms of two inefficiencies of exclusive bank–firm relationships.[2] First, according to the hold-up literature, sharing lending avoids the expropriation of informational rents and improves firms' incentives to make proper investment choices (for example, Rajan, 1992; Hellwig, 1991, 2000; and, in particular, Von Thadden, 1992, 2004). Second, multiple-bank lending helps with the soft-budget-constraint problem in that it enables banks not to extend further inefficient credit, thus reducing firms' strategic defaults (Dewatripont and Maskin, 1995; Bolton and Scharfstein, 1996). Both of these theories consider multiple-bank lending as a way to improve entrepreneurs' incentives, and focus on firms' decisions to borrow from more than one bank. Neither of them, however, addresses how multiple-bank lending affects banks' incentives to monitor, and thus cannot explain the apparent discrepancy between the empirical observation of multiple bank relationships and the importance of bank monitoring. In contrast, we analyze the incentives of multiple monitors, and show that multiple-bank lending can be compatible with the monitoring role of banks.

Other papers have analyzed the role of banks as monitors in explaining various features of relationship banking. Besanko and Kanatas (1993) focus on banks' incentives to monitor to justify the coexistence of banks and capital markets in a context where only one bank operates and monitors. Carletti (2004) analyzes why firms may prefer to borrow from multiple lenders when individual-bank lending leads to an overmonitoring problem. Millon and Thakor (1985) focus on the trade-off between better information production and the free-riding problem that arises as the intermediary increases in size, thereby explaining why financial intermediaries are of finite size. Winton (1995) models the impact of portfolio diversification and capitalization on banks' monitoring incentives in order to explain banks' finite size and analyze the welfare effects of different intermediated equilibria. In contrast to these papers, we analyze how the number of bank

[1] The focus on SMEs also has the advantage that banks' decisions to enter into multiple relationships are very unlikely to be driven by regulatory limits on bank exposures to individual firms (see, for example, Berger et al., 2005).

[2] Other explanations for multiple bank relationships include firms' desire to reduce overmonitoring problems and the liquidity risk affecting exclusive bank–firm relationships (Carletti, 2004; Detragiache et al., 2000).

relationships affects banks' monitoring incentives when banks have limited lending capacities, and greater diversification helps reduce the agency problem vis-à-vis depositors. Our focus is on the monitoring and funding roles of financial intermediaries as an explanation for the use of a limited number of multiple-banking relations. In this respect, the paper extends the analysis in Cerasi and Daltung (2000) by analyzing multiple-bank lending in terms of greater diversification and better monitoring incentives.[3]

Finally, the paper shares some insights with the literature on financial structure as a commitment to monitor. As in Holmstrom and Tirole (1997), Chiesa (2001) and Almazan (2002) we focus on the importance of limited lending capacities, but enrich the framework by introducing multiple monitors and diversification opportunities. These features link the paper also to Thakor (1996), who analyzes firms' incentives to borrow from multiple banks as a way to reduce the probability of being credit rationed. In contrast, we analyze different lending structures in a context where banks perform postlending monitoring and multiple-bank relationships is not always profitable for banks.

The remainder of the paper is organized as follows. Section 2 describes the basic model. Section 3 analyzes the equilibrium with individual-bank lending, and Section 4 presents the one with multiple-bank lending. Section 5 compares the two equilibria and discusses the lending structure with higher expected profits for banks. Section 6 extends the basic model. Section 7 discusses the robustness of the analysis. The empirical predictions of the model are contained in Section 8, and concluding remarks are in Section 9. All proofs are in Appendix C.

2. The basic model

Consider a two-date economy ($T = 0, 1$) with two banks, numerous firms and investors. Banks have one unit of funds each, and extend loans. Firms have access to a risky investment project, and need external funds to finance it. They compete to attract bank funds and only two of them at most obtain financing.

Projects are risky and their returns are i.i.d. across firms. Each project $i \in \{1, 2\}$ requires 1 unit of indivisible investment at date 0, and yields a return $X_i = \{0, R\}$ at date 1. The success probability of project i, $p_i = \Pr\{X_i = R\}$, depends on the behavior of its entrepreneur. It is p_H if he behaves well, and p_L if he misbehaves, with $p_H > p_L$. Misbehavior renders entrepreneurs a non-transferable private benefit B, which can be thought of as a quiet life, managerial perks, and diversion of corporate revenues for private use. There is a moral hazard problem because entrepreneurs' behavioral choices are not observable.

Banks have an amount of inside equity E and raise an amount of deposits $D = 1 - E$ so that they have limited loanable funds equal to $D + E = 1$. With individual-bank lending each bank lends to one firm only, while with multiple-bank lending banks share lending and each of them finances two firms. (We relax this assumption in Section 6 below.) In either case banks cannot perfectly diversify, but financing two firms allow them to achieve a better degree of diversification than financing only one, for given total loanable funds.

Banks extend loans to entrepreneurs if they expect non-negative profits, i.e., if they expect a return at least equal to the gross return $y \geqslant 1$ from an alternative investment. To provide a role for bank monitoring, we assume that lending without monitoring is not feasible. Specifically, we

[3] A contrasting view is in Winton (1999), where diversification may worsen banks' incentives to monitor and increase their chance of failure when loans are sufficiently exposed to sector downturns.

proceed under the assumptions:

$$p_H R > y > p_L R + B, \tag{A1}$$

and

$$\Delta\left(R - \frac{y}{p_H}\right) < B, \tag{A2}$$

where $\Delta = p_H - p_L$. Assumption (A1) means that projects are creditworthy only if firms behave well. Assumption (A2) entails that the private benefit B is sufficiently high to induce firms to misbehave even when loan rates are set at the lowest possible level $\frac{y}{p_H}$ that makes banks break even. Said differently, (A2) implies that firms cannot be given monetary incentives to behave well. Given (A1) and (A2), bank monitoring is essential for project financing to take place, since otherwise firms misbehave and banks make negative profits.

Bank monitoring allows banks to detect and prevent entrepreneurial misbehavior thus increasing the success probability of the project. Each bank $j \in \{1, 2\}$ chooses to monitor project i with an effort $m_{ij} \in [0, 1]$, which corresponds to the probability with which bank j makes firm i behave well.[4] Monitoring is costly; an effort m_{ij} costs $C(m_{ij}) = \frac{c}{2} m_{ij}^2$. The convex cost function reflects the greater difficulty for a bank to find out more and more about a firm and control entrepreneurial action; and it implies diseconomies of scale in monitoring. The size of the monitoring costs is determined by the parameter c (henceforth, also referred to as the cost of monitoring).

Banks' monitoring efforts are not observable to either investors or other banks. This introduces another moral hazard problem in the model. Banks choose the amount of monitoring to maximize their expected profits, and the equilibrium monitoring level depends on the number of firms (or projects) banks finance.

The timing of the model is as follows. At the beginning of date 0 each bank j sets a deposit rate r_j. Investors deposit their funds and the banks use these funds to make loans to firms. Then each bank chooses the effort m_{ij} with which to monitor project i. At date 1 project returns are realized and claims are settled. Figure 1 summarizes the timing of the model.

We solve the model by analyzing firstly the equilibrium with individual-bank lending and secondly the equilibrium with multiple-bank lending. Then we compare the equilibria in the two lending structures, and consider which one leads to higher expected profits for banks.

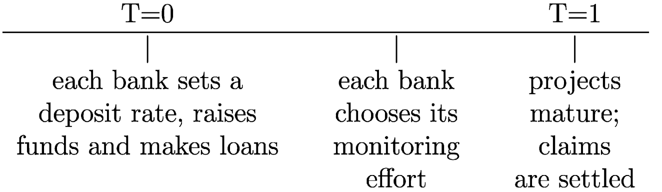

Fig. 1. Timing of the model.

[4] The monitoring technology builds on Besanko and Kanatas (1993) in that monitoring induces entrepreneurs to behave well. An alternative possible interpretation is that banks can provide some valuable advice to firms about how to run their investment projects, thus increasing their success probabilities.

3. Individual-bank lending

We start by characterizing the equilibrium with individual-bank lending (IL). Each bank j sets the deposit rate r_j to give depositors a return at least equal to the alternative return y and finances one firm. Then it chooses the monitoring effort m_{ij} that maximizes its expected profit. Since banks act independently of each other and behave symmetrically, we focus for simplicity on a single representative bank and a single firm, thus avoiding subscripts.

The bank's expected profit can be expressed as

$$\pi = E\max\{X - rD, 0\} - yE - \frac{c}{2}m^2, \tag{1}$$

where the first term represents the expected return from the project after depositors have been repaid, the second term is the opportunity cost of bank equity, and the third term is the monitoring cost.

As expression (1) shows, the deposit contract carries a bankruptcy risk. Since the bank is subject to limited liability and grants risky loans, depositors may not obtain the promised deposit rate. The probability of them being repaid depends on the success probability of the project, which is given by

$$p = mp_H + (1-m)p_L = p_L + m\Delta,$$

where m is the probability of (successfully) detecting entrepreneurial misbehavior. Thus, the higher m, the higher the project success probability, and the more the bank can honor its repayment obligation. We can then rewrite (1) as

$$\begin{aligned}\pi &= E(X) - rD + E\max\{rD - X, 0\} - yE - \frac{c}{2}m^2\\ &= pR - [r - S]D - yE - \frac{c}{2}m^2,\end{aligned} \tag{2}$$

where $[r-S]D = rD - E\max\{rD - X, 0\}$ is the expected return to depositors, and $S = (1-p)r$ is the per-unit expected shortfall on the deposit contract. Since there is an excess demand for bank credit, firms compete away their pecuniary returns from the projects to attract funds and banks retain all the surplus R.

Proposition 1 characterizes the individual-bank lending case.

Proposition 1. *The equilibrium with individual-bank lending, in which each bank monitors each project with effort m^{IL} and offers the deposit rate r^{IL}, is characterized by the solution to the following equations*:

$$\Delta R + \frac{\partial S^{\mathrm{IL}}}{\partial m^{\mathrm{IL}}}D - cm^{\mathrm{IL}} = 0, \tag{3}$$

$$r^{\mathrm{IL}} - S^{\mathrm{IL}} = y, \tag{4}$$

where $\frac{\partial S^{\mathrm{IL}}}{\partial m^{\mathrm{IL}}} = -\Delta r^{\mathrm{IL}}$.

Proposition 1 shows the importance of bank monitoring in the model. Monitoring makes lending feasible, and banks have an incentive to always exert a positive effort since the marginal unit at 0 is costless ($C'(0) = 0$). As already mentioned, however, raising deposits implies the well-known moral hazard problem of external financing. A higher monitoring level benefits both the bank and depositors, as it reduces the bankruptcy risk of the deposit contract and consequently the expected shortfalls. Since only the bank incurs the cost of monitoring and the deposit rate is set before monitoring is decided, its incentives are reduced and decrease with the size of expected shortfalls. The second term in (3), $\frac{\partial S^{\mathrm{IL}}}{\partial m^{\mathrm{IL}}}D$, captures this incentive mechanism. This term has a negative sign indicating that lower monitoring increases both the expected shortfalls and their derivative. The size of the expected shortfalls depends on the (exogenous) distribution of the return of the project X and the level of monitoring m. This suggests an interrelation between monitoring and expected shortfalls. On the one hand, the lower the expected shortfalls—that is, the lower the variance of the distribution of X—the higher the monitoring m in equilibrium. On the other hand, the higher m the lower the expected shortfalls and their derivative.

Bank monitoring affects also the deposit rate r^{IL} in (4) through the size of the expected shortfalls S^{IL}. The bank sets the per-unit deposit rate at the lowest level which induces investors to deposit their funds. The value of r^{IL} rises when the bank exerts a low level of monitoring, since depositors have to be compensated for the higher expected shortfalls. This in turn increases S^{IL} and $\frac{\partial S^{\mathrm{IL}}}{\partial m^{\mathrm{IL}}}$, thus reducing monitoring even further.

The severity of banks' moral hazard problem depends on the level of inside equity E (or alternatively, the amount of deposits D), the project return R, and the cost of monitoring c. A high level of E (or a small D) improves banks' incentives to monitor and decreases the expected shortfalls. This effect is well known (for example, Jensen and Meckling, 1976). Reducing the amount of external financing allows banks to benefit more from their monitoring thus improving their incentives. The same happens for a high R or a low c because they increase the marginal benefit banks appropriate from monitoring. Thus, the equilibrium monitoring effort m^{IL} grows with the amount of inside equity and the project return, whereas it falls with the cost of monitoring.

4. Multiple-bank lending

We now turn to the equilibrium with multiple-bank lending (ML). As before, the equilibrium requires that each bank j sets the deposit rate r_j to satisfy investors' participation constraints, and that it chooses the monitoring effort m_{ij} for each project i so as to maximize its expected profit.

The difference with the individual-bank lending case is that now the two banks finance both firms and interact in their monitoring decisions. We assume that each bank lends a half unit to each firm and obtains a return of $\frac{R}{2}$ per project in the case of success. Since there is an excess demand for bank credit and banks have limited lending capacities, they still extract the surplus from the entrepreneurs and share the full project return in the case of success. Banks choose how much to monitor each project simultaneously and, given the non-observability of their efforts, also non-cooperatively. Their efforts are however interrelated in the impact on the success probability of the project. It is enough that one bank detects misbehavior to increase the success probability of the whole project. The idea is that monitoring delivers a public good, and all banks financing a firm benefit from the higher success probability of the project.

Given these considerations, the total monitoring effort (or probability of detection) that the two banks exert in project i is

$$M_i = 1 - (1 - m_{ij})(1 - m_{i,-j}), \tag{5}$$

and the success probability of project i is

$$p_i = M_i p_H + (1 - M_i) p_L = p_L + M_i \Delta, \tag{6}$$

with $i, j \in \{1, 2\}$ and $j \neq -j$. Since each bank j finances two (independent) projects, it has a (total) return from the loans given by $Z = \frac{X_i + X_{-i}}{2}$ with $i \neq -i$ and expected profit equal to

$$\pi_j = E \max\{Z - r_j D, 0\} - yE - \frac{c}{2} \sum_{i=1}^{2} m_{ij}^2. \tag{7}$$

Similarly to before, the first term in (7) represents the expected return from the two projects that bank j finances net of depositors' repayment, the second term is the opportunity cost of equity, and the third term is the total cost of monitoring the two projects.

As with individual-bank lending, the deposit contract carries a bankruptcy risk because banks may not be able to repay depositors in full. The size of such risk is different now because of the new distribution of the return of loans and of the different monitoring level banks will exert in equilibrium. Again we can define $[r_j - S]D = r_j D - E \max\{r_j D - Z, 0\}$ as the expected return to depositors, with S being the per-unit expected shortfall, and express (7) as

$$\pi_j = \sum_{i=1}^{2} p_i \left(\frac{R}{2}\right) - [r_j - S]D - yE - \frac{c}{2} \sum_{i=1}^{2} m_{ij}^2 \tag{8}$$

(see Appendix A for a full derivation of (8) and the exact expression for S).

Proposition 2 characterizes the equilibrium with multiple-bank lending.

Proposition 2. *The equilibrium with multiple-bank lending is unique and symmetric. Each bank monitors each project with effort $m_{ij} = m^{\mathrm{ML}}$ and offers the deposit rate $r_j = r^{\mathrm{ML}}$, where m^{ML} and r^{ML} solve the following equations*:

$$\frac{\Delta R}{2}\left(1 - m^{\mathrm{ML}}\right) + \frac{\partial S^{\mathrm{ML}}}{\partial m^{\mathrm{ML}}} D - c m^{\mathrm{ML}} = 0, \tag{9}$$

$$r^{\mathrm{ML}} - S^{\mathrm{ML}} = y. \tag{10}$$

Comparing Eq. (9) with (3) shows how banks' equilibrium monitoring efforts with multiple-bank lending differ from those with individual-bank lending. On the one hand, banks finance two independent projects and reach a greater degree of diversification than with individual-bank lending for given equal total loanable funds. This enhances banks' incentives via a reduction of the variance of the distribution of the return of the loans Z and thus of the expected shortfalls S^{ML} and their derivative (their exact expression is contained in the proof of the proposition). On the other hand, since monitoring is privately costly and banks do not coordinate in their choices, multiple-bank lending entails free-riding and duplication of effort, which in turn curtail banks' incentives as reflected in the term $\frac{1}{2}(1 - m^{\mathrm{ML}})$. The equilibrium monitoring effort m^{ML} balances these contrasting effects.

The equilibrium with individual and multiple-bank lending differs also in terms of deposit rates. As Eqs. (10) and (4) show, the difference in deposit rates depends on the expected shortfalls S^{ML} and S^{IL}, and thus again on the interrelations between greater diversification, free-riding and duplication of effort. As before, the deposit rate has an indirect effect in equilibrium on the amount of monitoring banks exert through the term $\frac{\partial S^{\mathrm{ML}}}{\partial m^{\mathrm{ML}}}$ in (9). Thus, the higher r^{ML}, the higher is S^{ML} and the lower is m^{ML}.

To sum up, the non-observability of monitoring efforts and the lack of cooperation in banks' choices implies a trade-off with multiple-bank lending in terms of monitoring incentives between greater diversification, free-riding and duplication of effort. Clearly, if banks could cooperate, sharing lending would only imply greater diversification and would always lead to higher individual monitoring and expected profits than individual-bank lending.[5] As we will see in the next section, however, even in the absence of cooperation multiple-bank lending may imply higher per-project monitoring than individual-bank lending although the individual monitoring efforts may be lower.

5. Comparing individual-bank and multiple-bank lending

We now turn to the comparison of banks' equilibrium expected profits with individual-bank and multiple-bank lending to determine the optimal lending structure. Once we substitute $D + E = 1$ and the respective equilibrium monitoring efforts and deposit rates in (2) and (8), we can express banks' expected profits as

$$\pi^{\mathrm{IL}} = p^{\mathrm{IL}} R - y - \frac{c}{2}\big(m^{\mathrm{IL}}\big)^2, \tag{11}$$

$$\pi^{\mathrm{ML}} = p^{\mathrm{ML}} R - y - c\big(m^{\mathrm{ML}}\big)^2, \tag{12}$$

if they lend individually or share lending, respectively. In both expressions the terms represent, in order, the expected return from the projects each bank finances, the return from the alternative investment—which is equal from (4) and (10) to the expected repayment to depositors—and the total monitoring costs.

Whether multiple-bank lending leads to higher expected profits than individual-bank lending depends on the relative differences between the per-project success probabilities—and therefore the per-project monitoring efforts—and the total monitoring costs in (11) and (12). Given the complex analytical expressions for the equilibrium monitoring efforts and the expected shortfalls, we cannot directly compare these two expressions. We then proceed in two steps. We first compare the per-project monitoring efforts with individual and multiple-bank lending, and study how they interrelate with the monitoring costs in determining banks' expected profits. Then, we analyze which lending structure leads to higher expected profits for banks. We start with the following result.

Proposition 3. *There exists a value $\overline{m} \in (0,1)$ such that the per-project monitoring effort with multiple-bank lending is higher than with individual-bank lending ($M^{\mathrm{ML}} > m^{\mathrm{IL}}$) if the individual monitoring effort is $m^{\mathrm{ML}} > \overline{m}$.*

Proposition 3 shows that the benefit of greater diversification attainable with multiple-bank lending may be important enough to achieve greater per-project monitoring than with individual-bank lending. To see when this result occurs, we conduct some comparative statics.

Proposition 4. *The threshold $\overline{m} \in (0,1)$ increases with the amount of inside equity E and the project return R, while it decreases with the cost of monitoring c.*

[5] Note also that, even if cooperation was possible, banks would not find it optimal to delegate the monitoring task to one of them because of convex monitoring costs. More importantly, delegation is not feasible in our context because banks get symmetric rewards from monitoring.

The basic intuition behind Proposition 4 is that multiple-bank lending leads to higher per-project monitoring efforts than individual-bank lending when banks' moral hazard problem is severe enough. If banks exert a low level of monitoring when lending individually, the greater diversification attainable with multiple-bank lending has a significant marginal impact on banks' monitoring incentives and may dominate the drawbacks of free-riding and duplication of effort. By contrast, if banks exert a high level of monitoring when lending individually, free-riding and duplication of effort are likely to dominate and lead to lower per-project monitoring in the case of multiple-bank lending. As a consequence, the threshold $\overline{m}$ depends on the severity of the banks' moral hazard problem, which is decreasing with E and R and increasing with c.

An important implication of this discussion is that the incentive mechanism of the greater diversification achievable with multiple-bank lending works only if banks raise deposits, i.e., if they are leveraged. We have the following

Corollary 1. *Multiple-bank lending always leads to lower per-project monitoring effort than individual-bank lending* ($M^{\mathrm{ML}} < m^{\mathrm{IL}}$) *if banks do not raise deposits* ($E = 1$).

When banks do not raise deposits, there is no benefit of greater diversification on depositors' expected shortfalls. This suggests that in our model banks benefit from greater diversification only if this helps reduce the agency problem with depositors and not as a simple risk sharing mechanism.

Given the previous results, we now analyze how per-project monitoring efforts and total costs contribute to the determination of the optimal lending structure.

Lemma 1. *Higher per-project monitoring effort* ($M^{\mathrm{ML}} > m^{\mathrm{IL}}$) *is necessary for higher banks' expected profits with multiple-bank lending* ($\pi^{\mathrm{ML}} > \pi^{\mathrm{IL}}$) *if* $m^{\mathrm{ML}} > \frac{m^{\mathrm{IL}}}{\sqrt{2}}$*, and is sufficient otherwise.*

Lemma 1 hinges on the interaction between the convexity of the monitoring cost function and the duplication of effort due to the lack of coordination in banks' monitoring choices with multiple-bank lending. If banks individually exert a level of monitoring higher than $\frac{m^{\mathrm{IL}}}{\sqrt{2}}$ when they share lending, they incur higher total costs because the duplication of effort dominates the convexity of the cost function. In this case, multiple-bank lending leads to higher expected profits for banks only if the higher per-project monitoring effort suffices to dominate the higher costs. In contrast, when $m^{\mathrm{ML}} \leqslant \frac{m^{\mathrm{IL}}}{\sqrt{2}}$, the convexity dominates in lowering total costs and banks always have greater expected profits if sharing lending leads to a greater per-project monitoring.

Lemma 1 implies various combinations of per-project monitoring and total monitoring costs in determining the profitability of the two lending structures. We demonstrate the relevance of the various combinations graphically as a function of the parameters E, R and c. Figure 2 depicts the curves where banks' expected profits (π^{IL} and π^{ML}), per-project monitoring (m^{IL} and M^{ML}) and total monitoring costs (C^{IL} and C^{ML}) are the same in the two lending structures as a function of the cost of monitoring c and the project return R when banks raise only deposits ($E = 0$).[6] Multiple-bank lending implies higher expected profits, higher per-project monitoring and lower total costs than individual-bank lending below the respective curve; while the opposite happens above each curve.

[6] The other parameters of the model are fixed to $p_H = 0.8$, $p_L = 0.6$, and $y = 1$.

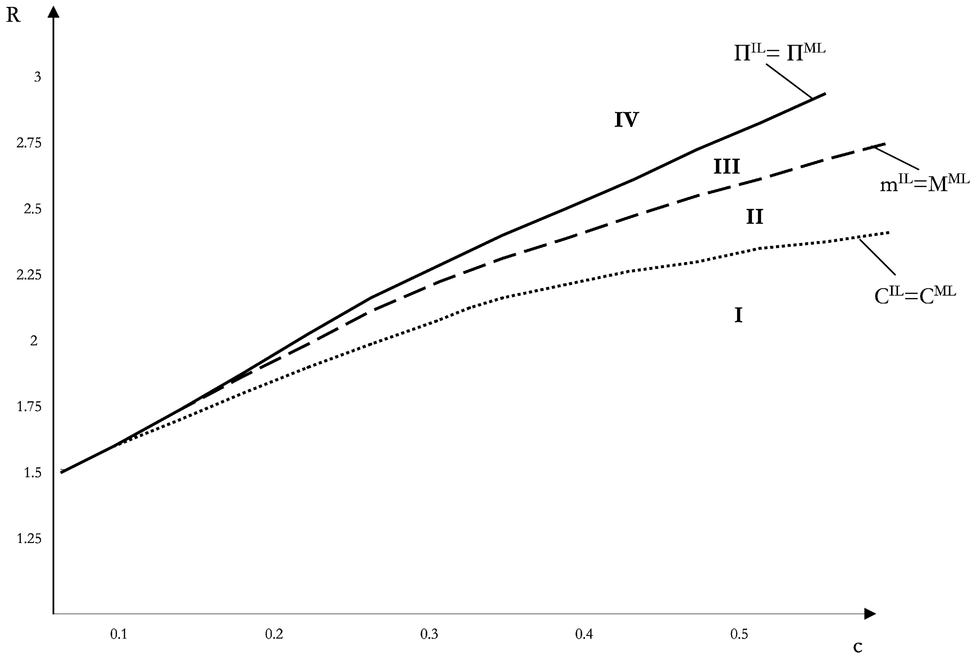

Fig. 2. Banks' expected profits, per-project monitoring and total monitoring costs with individual and multiple-bank lending. The figure shows the curves where banks' expected profits, per-project monitoring and total costs with individual lending (Π^{IL}, m^{IL} and C^{IL}) are equal to those with multiple-bank lending (Π^{ML}, M^{ML} and C^{ML}) as functions of the monitoring cost c and the project return R. The figure shows four areas: I where $\Pi^{\mathrm{ML}} > \Pi^{\mathrm{IL}}$, $M^{\mathrm{ML}} > m^{\mathrm{IL}}$ and $C^{\mathrm{ML}} > C^{\mathrm{IL}}$; II where $\Pi^{\mathrm{ML}} > \Pi^{\mathrm{IL}}$, $M^{\mathrm{ML}} > m^{\mathrm{IL}}$ and $C^{\mathrm{ML}} < C^{\mathrm{IL}}$; III where $\Pi^{\mathrm{ML}} > \Pi^{\mathrm{IL}}$, $M^{\mathrm{ML}} < m^{\mathrm{IL}}$ and $C^{\mathrm{ML}} < C^{\mathrm{IL}}$; IV where $\Pi^{\mathrm{ML}} < \Pi^{\mathrm{IL}}$, $M^{\mathrm{ML}} < m^{\mathrm{IL}}$ and $C^{\mathrm{ML}} < C^{\mathrm{IL}}$. The figure is drawn for success probabilities of the project $p_H = 0.8$ and $p_L = 0.6$, alternative return $y = 1$, and deposits $D = 1$.

The graph highlights four areas. Multiple-bank lending leads to higher expected profits for banks in all areas except IV, but the determinants of its higher profitability differ across the areas. Area I depicts the case where multiple-bank lending is more profitable because higher per-project monitoring ($M^{\mathrm{ML}} > m^{\mathrm{IL}}$) dominates higher costs ($C^{\mathrm{ML}} > C^{\mathrm{IL}}$). Area II shows the case where higher per-project monitoring suffices because costs are lower ($C^{\mathrm{ML}} < C^{\mathrm{IL}}$). Finally, in area III the convexity prevails in reducing total costs so that banks have higher expected profits despite exerting lower per-project monitoring. Overall, the figure shows the importance of per-project monitoring. Except in area III, the lending structure with higher per-project monitoring is always more profitable. As a consequence, the higher profitability of multiple-bank lending reflects the behavior of per-project monitoring, which, as shown in Proposition 4, decreases with the return of the project R and increases with the cost of monitoring c.

To complete the comparative statics, we also analyze how the profitability of the two lending structures changes with the amount of inside equity E. Figure 3 depicts the curves where banks' expected profits are the same with individual and multiple-bank lending ($\pi^{\mathrm{IL}} = \pi^{\mathrm{ML}}$) as a function of the cost of monitoring and the return of the project when the amount of inside equity varies from $E = 0$ to $E = 0.2$ (i.e., from $D = 1$ to $D = 0.8$).

The figure shows that the attractiveness of multiple-bank lending decreases when banks are more equity financed. Whereas sharing lending is more profitable in areas I and II when $E = 0$, it is no longer so in area II as E increases. The intuition is as before. As shown in Proposition 4,

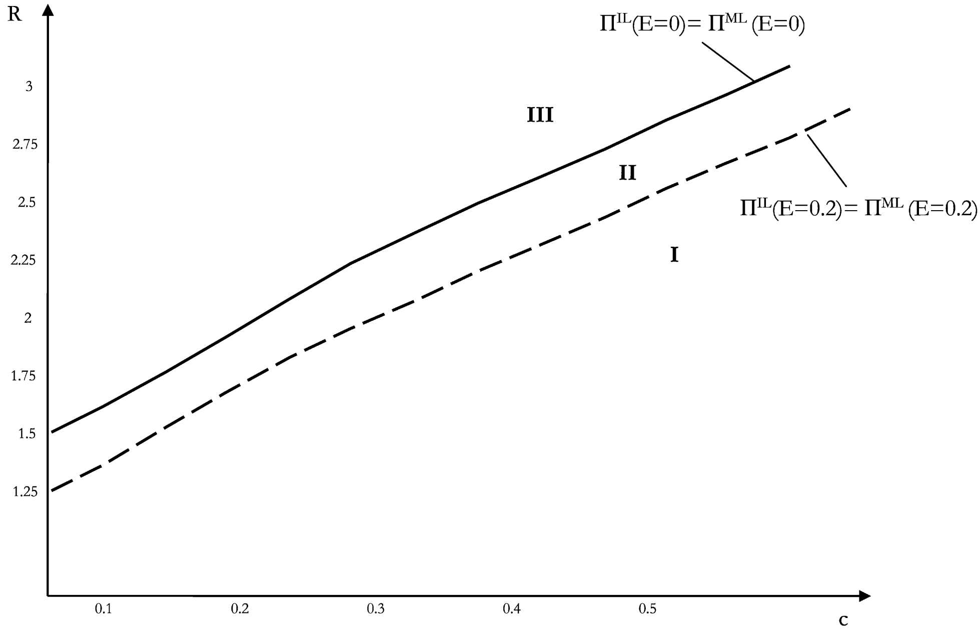

Fig. 3. Banks' expected profits with individual and multiple-bank lending. The figure shows the curves where banks' expected profits with individual lending Π^{IL} are equal to those with multiple-bank lending Π^{ML} for different values of inside equity ($E = 0$ and $E = 0.2$) as functions of the monitoring cost c and the project return R. The figure shows three areas: I where $\Pi^{\mathrm{ML}} > \Pi^{\mathrm{IL}}$ for both $E = 0$ and $E = 0.2$; II where Π^{ML} $(E = 0) > \Pi^{\mathrm{IL}}$ $(E = 0)$ but Π^{ML} $(E = 0.2) < \Pi^{\mathrm{IL}}$ $(E = 0.2)$; III where $\Pi^{\mathrm{ML}} < \Pi^{\mathrm{IL}}$ for both $E = 0$ and $E = 0.2$. The figure is drawn for success probabilities of the project $p_H = 0.8$ and $p_L = 0.6$, and alternative return $y = 1$.

a larger fraction of inside equity reduces banks' moral hazard thus increasing the threshold $\overline{m}$. This reduces the range of c and R where per-project monitoring is higher than with individual-bank lending, thus also reducing the range of parameters where multiple-bank lending is more profitable.

To summarize these results[7]:

Proposition 5. *Multiple-bank lending leads to higher banks' expected profits than individual-bank lending when the amount of inside equity E and the project return R are low, and the cost of monitoring c is high. Individual-bank lending leads to higher banks' expected profits otherwise.*

6. Extensions

The essential idea behind multiple-bank lending is that banks have limited loanable funds that constrain the degree of diversification (and thus the level of monitoring) they can achieve as individual lenders. So far we have considered limits of a "fixed size" in that banks have one

[7] The result that the attractiveness of multiple-bank lending decreases with the project return R suggests that the fact that banks obtain the full return from lending is not crucial. Assuming that banks obtain lower loan rates would simply correspond to a reduction of R and would then not modify our qualitative results.

unit of funds each and can finance either one project as individual lenders or two projects when sharing lending. We now depart from this simple set up in two ways. First, we allow banks to increase their size by raising more deposits and finance the same number of projects with individual and multiple-bank lending. Second, we consider an economy with $k \geqslant 2$ banks and analyze how the benefit of greater diversification varies with the number of banks entering into multiple relationships.

6.1. *Leverage versus free-riding*

In the basic model we have constrained bank size to one unit and have analyzed multiple-bank lending as a way for banks to increase the number of projects they finance, for *given* total loanable funds. We now extend this framework by letting banks increase their size through more deposits when acting as individual lenders so as to finance the same number of projects as with multiple-bank lending.[8] This implies a new trade-off in the determination of the optimal lending structure. Rather than focusing on greater diversification and free-riding as in the basic model, we now focus on the trade-off between greater leverage and free-riding as ways to achieve a *given* (equal) level of diversification.

Consider the same economy as in the basic model. For a given amount of inside equity E each bank raises an amount of deposits $D_2 = 2 - E$ and finances two projects as an individual lender; or it raises $D_1 = 1 - E$ and shares with another bank the financing of two projects. Since $D_2 > D_1$, financing two projects as an individual lender implies higher leverage for the bank and thus, ceteris paribus, a more severe moral hazard problem in bank monitoring. Following the same analysis as in the basic model, we can express the bank's expected profit with individual-bank lending as

$$\pi = \sum_{i=1}^{2} p_i R - [r - S]D_2 - yE - \frac{c}{2}\sum_{i=1}^{2} m_i^2, \tag{13}$$

where the success probability is $p_i = p_L + m_i\Delta$, $[r - S]D_2 = rD_2 - E\max\{r_j D_2 - W, 0\}$ is the expected return to depositors, and $W = X_i + X_{-i}$ is the (total) return from the loans with $i \neq -i$. The terms in (13) have the usual meaning with the only difference that now individual-bank lending implies financing two projects rather than only one. Note also that, since banks act independently, we focus again on a single representative bank and avoid subscript j.

The equilibrium is now characterized as follows (the exact expressions for S^ℓ and $\frac{\partial S^\ell}{\partial m^\ell}$ are provided in the proof of the proposition in Appendix C).

Proposition 6. *The equilibrium with individual-bank lending, in which each bank finances two projects, monitors each project with effort $m_i = m^\ell$, and offers the deposit rate $r = r^\ell$, solves the following two equations*:

$$\Delta R + \frac{\partial S^\ell}{\partial m^\ell} D_2 - cm^\ell = 0, \tag{14}$$

$$r^\ell - S^\ell = y. \tag{15}$$

[8] We are grateful to an anonymous referee for suggesting this extension along the lines of Winton (1995).

The equilibrium in Proposition 6 resembles the one in Proposition 1, but the equilibrium values of monitoring and of the deposit rate are different because of the new distribution of the return of loans and the higher amount of leverage. To see how this new equilibrium with individual-bank lending contrasts to the one with multiple-bank lending, we compare it with that in Proposition 2. We have the following

Proposition 7. *There exists a value $\widehat{m} \in (0,1)$ such that the per-project monitoring effort with multiple-bank lending is higher than with individual-bank lending with two projects ($M^{\mathrm{ML}} > m^{\ell}$) if the individual monitoring effort is $m^{\mathrm{ML}} > \widehat{m}$. The threshold $\widehat{m}$ increases with the amount of equity E.*

Proposition 7 shows that multiple-bank lending represents a better way to achieve greater diversification and obtain higher per-project monitoring when banks have a small amount of inside equity E; while individual-bank lending with higher leverage is better otherwise. The intuition is simple. When E is small, banks have to raise a large fraction of deposits in order to be able to finance as many projects as with multiple-bank lending. This increases ceteris paribus their moral hazard problem and leads to lower per-project monitoring than with multiple-bank lending. The opposite happens when E is large.

6.2. A larger number of banks

We now extend the basic model by allowing a number of banks $k \geqslant 2$ in the economy. This allows us to analyze how the benefit of greater diversification varies with the number of banks entering into multiple relationships; and to provide a justification for the empirical observation that multiple-bank relationships often consist of many banks.

Consider $k \geqslant 2$ banks with total loanable funds $D + E \geqslant 1$. One way to think about it is that banks have a fixed amount of inside equity E and are subject to a capital constraint $\frac{1}{\beta}$ (with $\beta > 1$), which limits the amount they can lend to $D + E = \beta E$.[9] Banks finance then either $(D + E)$ firms as individual lenders or share financing and lend $\frac{1}{k}$ to each of $k(D + E)$ projects. The rest of the model is as before; and for the sake of brevity we solve it directly as a function of k.

Given this, we can express the total monitoring banks exert on project i as

$$M_i = 1 - \prod_{j=1}^{k} (1 - m_{ij}),$$

and the success probability of project i as

$$p_i = M_i p_H + (1 - M_i) p_L = p_L + M_i \Delta, \tag{16}$$

[9] The idea of binding capital requirements to justify banks' limited loanable funds is in line with Thakor (1996), who provides both theory and evidence that increases in capital requirements decrease banks' aggregate lending. Note that capital requirements are distorsive in our framework as they play no role other than preventing full diversification. They can however be justified in a more general framework where projects are also subject to common risk factors and fully diversified portfolios are still risky (see Chiesa, 2001).

with $i \in \{1, \dots, k(D+E)\}$ and $j \in \{1, \dots, k\}$. Following the same procedure as in the basic model, we can derive bank j's expected profit as

$$\pi_j = \sum_{i=1}^{k(D+E)} p_i \frac{R}{k} - [r_j - S]D - yE - \frac{c}{2} \sum_{i=1}^{k(D+E)} m_{ij}^2, \tag{17}$$

where $[r_j - S]D$ are the total expected shortfalls to depositors and S is the per-unit expected shortfall (Appendix B contains the full derivation of (17) and the exact expression for S in this case). We can then characterize the equilibrium.

Proposition 8. *The equilibrium is unique and symmetric. Each bank monitors each project with effort $m_{ij} = m(k)$ and offers the deposit rate $r_j = r(k)$, where $m(k)$ and $r(k)$ solve the following equations*:

$$\frac{\Delta R}{k}\big(1 - m(k)\big)^{k-1} + \frac{\partial S(k)}{\partial m(k)} D - cm(k) = 0, \tag{18}$$

$$r(k) - S(k) = y. \tag{19}$$

Proposition 8 shows how the trade-off involved in multiple-bank lending varies with the number of banks k financing the same firm. On the one hand, increasing k worsens free-riding and duplication of effort (the term $\frac{1}{k}(1 - m(k))^{k-1}$), and reduces further banks' monitoring incentives. On the other hand, a higher k allows banks to finance a greater number of projects when entering into multiple-bank lending (from $(D+E)$ to $k(D+E)$ with $k > 2$). This lowers the variance of the distribution of the loan returns and expected shortfalls by more relative to the case with $k = 2$, thus enlarging the impact of diversification on banks' incentives.[10]

Proposition 8 suggests that sharing lending with a number of banks $k > 2$ may eventually lead to higher per-project monitoring than individual-bank lending even when this is not the case for $k = 2$. Figure 4 provides an example of this instance by depicting how individual and per-project monitoring efforts $m(k)$ and $M(k)$ change as a function of the number of banks k when $E = 0.5$ and $\beta = 12$ (which corresponds to capital requirements of 8%).[11]

The example in Fig. 4 shows that the marginal benefit of greater diversification can increase faster than the drawbacks of free-riding and duplication of effort with k and lead to a higher per-project monitoring when k is sufficiently large. In more formal terms, this suggests that the term $|\frac{\partial S(k)}{\partial m(k)} D|$ in (18) capturing the incentive mechanism of greater diversification decreases with k. The question is how fast it decreases, and how it impacts the optimal number of monitors relative to the other terms in (18) representing the costs of free-riding and duplication of effort. We have the following intermediate result.

Lemma 2. *The term $|\frac{\partial S(k)}{\partial m(k)} D| \to 0$ as $k \to \infty$.*

Lemma 2 suggests that sharing lending with an infinite number of banks would allow banks to achieve full diversification and eliminate the bankruptcy risk embodied in the deposit contract. However, banks may choose not to do it.

[10] Note that for $k = 1$ and $k = 2$ the equilibrium reduces to the same as that described in Propositions 1 and 2, respectively, if also $D + E = 1$.

[11] The other parameters of the model are $R = 1.52$, $c = 0.35$, $p_H = 0.8$, $p_L = 0.6$, and $y = 1$. Note also that now we approximate the distribution of the returns of banks' portfolios with a Normal distribution.

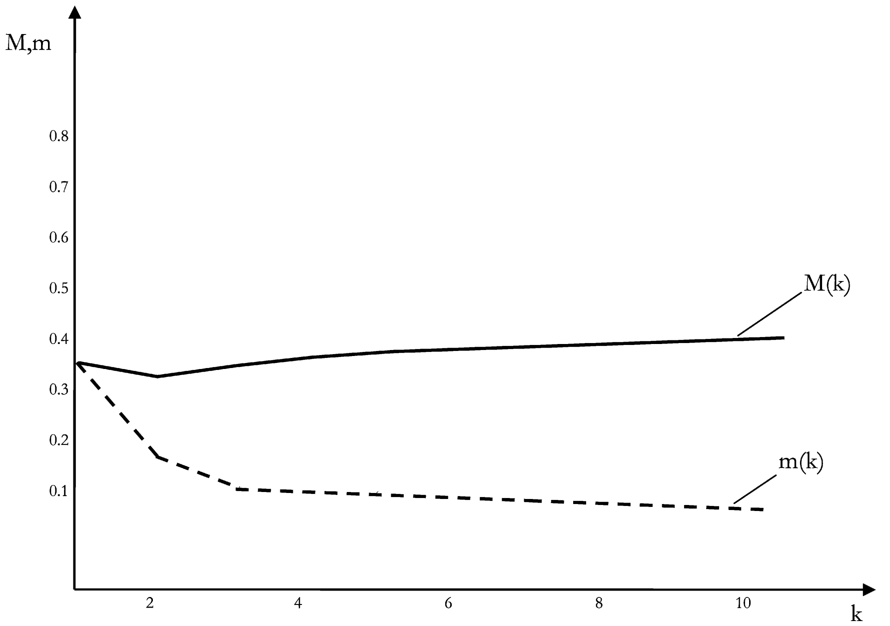

Fig. 4. Individual and per-project total monitoring efforts. The figure shows how the individual monitoring effort $m(k)$ and the per-project total monitoring effort $M(k)$ change as a function of the number of banks k. The figure is drawn for inside equity $E = 0.5$, project return $R = 1.52$, cost of monitoring $c = 0.35$, capital requirement equal to 8%, success probabilities of the project $p_H = 0.8$ and $p_L = 0.6$, alternative return $y = 1$.

Proposition 9. *Banks find it optimal to enter into multiple relationships with a finite number of banks.*

Differently from the basic model, banks now have the possibility to increase the number of banks with which they share financing and achieve full diversification. However, they do not find it optimal to do so. The optimal number of bank relationships is finite since, as k increases, the costs in terms of free-riding and duplication of effort eventually dominate the benefit of greater diversification. This result also suggests once again that in our model diversification is not beneficial in terms of risk sharing but only as a way to improve banks' monitoring incentives. When the limit to this benefit is reached, diversifying further is no longer desirable.

7. Discussion

In this section we analyze various aspects of our model. Specifically, we discuss banks' limited lending capacities, other diversification opportunities, and alternative monitoring technologies.

7.1. Limits to diversification

The key idea behind the optimality of multiple-bank lending is that banks have limited diversification opportunities and cannot fully diversify when lending individually. As one way to justify this, we have so far assumed that banks have limited loanable funds. More generally though, any situation which constrains banks' diversification opportunities is consistent with our theory. Examples are restrictions on banks' geographical scope and sector specialization. There is evidence that lending to firms situated in distant locations can be more costly because of information problems, transportation costs, and, if situated in foreign regions, differences in legal systems, supervisory regimes, corporate governance, language and cultural conditions (for example, Acharya et al., 2006). All these factors may limit the number of projects that banks can profitably finance as individual lenders, and thus, as in our model, leave scope for multiple-bank lending.

7.2. Alternative diversification opportunities

So far we have considered how banks can achieve better diversification by sharing lending with other banks or by increasing leverage. In practice, however, there are other ways of doing it. The most immediate is raising outside equity. This relaxes the limits on loanable funds, but it may be neither feasible (at least in the short term) nor optimal. Outside equity introduces in fact another agency problem between banks and equity holders, which reduces banks' incentives to monitor and is not ameliorated by greater diversification (Cerasi and Daltung, 2000). Moreover, as is well known from the corporate finance literature, raising outside equity is more costly in terms of foregone tax advantages, asymmetric information, and transaction costs than other forms of financing (for example, Jensen and Meckling, 1976; and numerous articles that have followed it). These considerations suggest that, even if allowed to raise outside equity, banks are likely to remain capital constrained and may still choose multiple-bank lending as a way to achieve better diversification.

Another way for banks to achieve greater diversification for given total loanable funds is to issue credit derivatives, such as credit default swaps. In our context, this implies the possibility for banks to act as single lenders and buy protection against borrowers' default at date 1 in exchange for a fixed initial fee. The effects of these instruments depend crucially on the identity of the seller, and the payment that the buyer receives in the case of default of some risky assets ("transfer"). If banks buy protection from dispersed investors, their monitoring incentives worsen. Like all forms of insurance, banks' incentives decrease in the size of the transfer they receive in the case of default.[12] By contrast, if banks exchange credit derivatives on their loans among each other, their monitoring incentives may improve. As with multiple-bank lending, all banks (both buyers and sellers of protection) monitor the risky projects underlying the credit derivatives, and, depending on the size of the transfer, they exert asymmetric or symmetric efforts. Thus, the exchange of credit derivatives allows banks to achieve levels of diversification, free-riding and duplication of efforts in between those attainable with individual and multiple-bank lending, and it leads to the same results as in each of them for extreme amounts of the transfer. Note however that credit derivatives are a relatively new innovation, and as such they are currently not available in all countries and for all types of banks.

7.3. Alternative monitoring technologies

The monitoring technology we have assumed so far gives banks a direct form of control on firms' behavior in that it allows banks to detect firms' project choices and intervene in case of

[12] The result may be different if there is aggregate risk, in which case (optimal) risk transfer instruments enhance monitoring incentives (see Chiesa, 2006).

misbehavior. Other forms of control are, however, plausible. For example, through monitoring banks could observe firms' behavior and liquidate them for a total value of C (for example, Rajan and Winton, 1995). Whether this leads to different results for the optimality of multiple-bank lending depends on how the liquidation value C is allocated among banks. Our qualitative results still hold if banks share C equally in case of default independently of whether they monitor. Results may differ, however, if a monitoring bank is the first to seize C. This reduces free-riding, but it may still reduce the attractiveness of multiple-bank lending if it leads to excessive duplication of effort.

8. Empirical implications

The main insight of the paper is to show that multiple-bank lending can be beneficial as it allows banks to increase the overall effort with which they monitor firms. This occurs when banks have low inside equity, the returns of firm projects are low and the cost of monitoring is high. The model thus has a number of empirical implications for the determinants of multiple-bank lending, some of which are consistent with recent empirical findings.

First, the attractiveness of multiple-bank lending should decrease as banks have more inside equity. If consolidation increases inside equity, at least in the short run, then the empirical findings of Degryse et al. (2004) that, post consolidation, banks are more likely to terminate lending relationships with firms borrowing from multiple banks, is suggestive of support for this prediction.

Second, the model predicts that multiple-bank lending should be optimal when banks lend to firms with low ex ante profitability, as is found by Detragiache et al. (2000), Petersen and Rajan (1994), Farinha and Santos (2002) and Guiso and Minetti (2006).

Third, multiple-bank lending should be more attractive when banks face high monitoring costs. The size of the monitoring cost relates to the ease with which banks can acquire information about firms and is therefore linked to the degree of firms' transparency as can be determined by disclosure and accounting standards. More broadly, monitoring costs can also be influenced by the degree of financial integration and of regulatory restrictions to the extent that these affect banks' ability to acquire information on firms operating in different sectors or geographical areas. Thus, the model predicts that banks should share lending when financing more opaque firms, and in sectors and/or countries with laxer accounting and disclosure standards, less integrated and more regulated markets. In line with this, Guiso and Minetti (2006) find that more informationally transparent firms use less multiple-bank lending as public information mitigates the costs banks have to incur in monitoring entrepreneurs.

9. Conclusion

This paper analyzes the optimality of multiple-bank relationships in a context where banks have limited diversification opportunities and are subject to a moral hazard problem in monitoring. Multiple-bank lending involves a trade off in terms of greater diversification, free-riding and duplication of effort; and leads to higher per-project monitoring than individual-bank lending whenever the benefit of greater diversification dominates. The attractiveness of multiple-bank lending decreases with the amount of banks' inside equity and firms' prior profitability, whereas it increases with the cost of monitoring.

The two important features of the analysis—leverage and limited diversification opportunities—capture two important aspects of the banking industry; and, together with the role of banks as monitors, make the analysis suitable for explaining the financing of small and medium businesses. In this respect, the paper departs from the existing theory of financial intermediation in suggesting that increasing the number of monitors may lead to higher overall effort when banks have limited diversification opportunities; and it provides an alternative to the hold-up and the soft-budget-constraint theories in explaining the optimality of multiple-bank relationships.

We develop the analysis under the assumption that all banks share financing equally when they enter into multiple-bank relationships. Allowing for asymmetric shares of financing would lead to results somewhere in between those obtained in the current analysis; and it may provide some useful insights into other important features of the banking systems such as the role of "housebanks," the functioning of syndicates or the use of credit derivatives. We leave these as avenues for future research.

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Appendix A. Banks' expected profits in the basic model with multiple-bank lending

The return of each bank's loans $Z = \sum_{i=1}^{2}(\frac{X_i}{2})$ has the Binomial distribution

$$Z = \begin{cases} 0 & (1-p_i)(1-p_{-i}), \\ \frac{R}{2} & p_i(1-p_{-i}) + p_{-i}(1-p_i), \\ R & p_i p_{-i}, \end{cases}$$

and average equal to

$$E(Z) = E\left(\frac{X_i + X_{-i}}{2}\right) = \frac{E(X_i) + E(X_{-i})}{2} = (p_i + p_{-i})\left(\frac{R}{2}\right), \tag{A.1}$$

where p_i is given by (6), $i \in \{1,2\}$ and $i \neq -i$. The expected profit of bank j is

$$\pi_j = E\max\{Z - r_j D, 0\} - yE - \frac{c}{2}\sum_{i=1}^{2} m_{ij}^2,$$

with $j \in \{1,2\}$. This can be rewritten using the transformation $\max\{0,x\} = x + \max\{0,-x\}$ as

$$\pi_j = E(Z) - r_j D + E\max\{r_j D - Z, 0\} - yE - \frac{c}{2}\sum_{i=1}^{2} m_{ij}^2.$$

This expression simplifies to (8) once we substitute (A.1) and $[r_j - S]D = r_j D - E\max\{r_j D - Z, 0\}$, where S is given by

$$S = r_j(1-p_i)(1-p_{-i}) + \frac{1}{D}\max\left\{r_j D - \frac{R}{2}, 0\right\}\left[p_i(1-p_{-i}) + p_{-i}(1-p_i)\right]. \tag{A.2}$$

Appendix B. Banks' expected profits for $k \geqslant 2$

Banks' expected profits for $k \geqslant 2$ are a generalization of the expressions in the basic model once we take into account that banks invest now in $k(D+E)$ projects. This implies that the return of each bank's loans $Z = \sum_{i=1}^{k(D+E)}(\frac{X_i}{k})$ is a sum of $k(D+E)$ random variables following a Binomial distribution with average:

$$E(Z) = E\left(\sum_{i=1}^{N}\left(\frac{X_i}{k}\right)\right) = \sum_{i=1}^{N}\frac{E(X_i)}{k} = \sum_{i=1}^{N} p_i\left(\frac{R}{k}\right), \tag{B.1}$$

where p_i is given by (16) and, as in the rest of the appendix, for brevity we use $N = k(D+E)$ so that $i \in \{1, \dots, N\}$. The expected profit of bank j is then

$$\pi_j = E\max\{Z - r_j D, 0\} - yE - \frac{c}{2}\sum_{i=1}^{N} m_{ij}^2,$$

which, using again the transformation $\max\{0, x\} = x + \max\{0, -x\}$, can be rewritten as

$$\pi_j = E(Z) - r_j D + E\max\{r_j D - Z, 0\} - yE - \frac{c}{2}\sum_{i=1}^{N} m_{ij}^2. \tag{B.2}$$

The expression can be further simplified to (17) once we substitute (B.1) and $[r_j - S]D = r_j D - E\max\{r_j D - Z, 0\}$, where S is given by

$$\begin{aligned} S = \frac{1}{D}\sum_{v=0}^{N}\max\left\{r_j D - v\frac{R}{k}, 0\right\}\Bigg[& p_i\binom{N-1}{v-1}(1-p_{-i})^{N-v}p_{-i}^{v-1} \\ & + (1-p_i)\binom{N-1}{v}p_{-i}^{v}(1-p_{-i})^{N-v-1}\Bigg], \end{aligned} \tag{B.3}$$

for $i \neq -i$, where v is the number of successful projects.

Appendix C. Proofs

Proof of Proposition 1. For a given r, the bank chooses m to maximize (2) with $S = (1-p)r$. The first-order condition gives (3), where $\frac{\partial S^{\mathrm{IL}}}{\partial m^{\mathrm{IL}}} = -\Delta r^{\mathrm{IL}}$. Setting $[r - S] = y$ after substituting m^{IL} gives (4). □

Proof of Proposition 2. For a given r_j, each bank j chooses m_{ij} to maximize (8). The first-order condition is given by

$$\frac{\partial \pi_j}{\partial m_{ij}} = (1 - m_{i,-j})\Delta\left\{(R - r_j D)p_{-i} + \max\left\{\frac{R}{2} - r_j D, 0\right\}(1 - 2p_{-i})\right\} - cm_{ij} = 0, \tag{C.1}$$

for $i, j \in \{1, 2\}$, $i \neq -i$ and $j \neq -j$. To see that there exists a unique equilibrium, we look at the second order condition as given by

$$\frac{\partial^2 \pi_j}{\partial m_{ij}^2} = -c < 0.$$

The negative sign of the second-order condition for any $m_{i,-j}$ indicates that banks' expected profits are globally concave, thus implying that the first-order conditions are binding in equilibrium. Furthermore, it is easy to derive from the ratio of the first-order conditions for project i $\frac{\partial \pi_j}{\partial m_{ij}} / \frac{\partial \pi_{-j}}{\partial m_{i,-j}} = 0$ that

$$(1 - m_{i,-j}) m_{ij} = (1 - m_{ij}) m_{i,-j}.$$

It follows that the unique equilibrium is symmetric. In this case, using $m_{ij} = m_{i,-j} = m^{\mathrm{ML}}$, $p_i = p_{-i} = p^{\mathrm{ML}}$, and $r_j = r_{-j} = r^{\mathrm{ML}}$, (A.2) simplifies to

$$S^{\mathrm{ML}} = r^{\mathrm{ML}} \left(1 - p^{\mathrm{ML}}\right)^2 + \frac{2}{D} \max\left\{ r^{\mathrm{ML}} D - \frac{R}{2}, 0 \right\} p^{\mathrm{ML}} (1 - p)^{\mathrm{ML}}. \tag{C.2}$$

We can then rearrange (C.1) as in (9), where

$$\frac{\partial S^{\mathrm{ML}}}{\partial m^{\mathrm{ML}}} D = \left(1 - m^{\mathrm{ML}}\right) \Delta \left\{ -r^{\mathrm{ML}} D \left(1 - p^{\mathrm{ML}}\right) + \max\left\{ r^{\mathrm{ML}} D - \frac{R}{2}, 0 \right\} \left(1 - 2p^{\mathrm{ML}}\right) \right\}. \tag{C.3}$$

Expression (10) follows from $[r - S] = y$ after substituting m^{ML}. □

Proof of Proposition 3. We compare m^{IL} and M^{ML} in equilibrium. To do this, we substitute the investors' individual rationality constraints in the respective first order conditions for the monitoring efforts, and we then compare them.

With individual-bank lending, we substitute (4) in (3) with $S^{\mathrm{IL}} = (1 - p^{\mathrm{IL}}) r^{\mathrm{IL}}$, and we rewrite it as

$$\Delta \left(R - \frac{yD}{p^{\mathrm{IL}}} \right) = c m^{\mathrm{IL}}, \tag{C.4}$$

where $p^{\mathrm{IL}} = p_L + m^{\mathrm{IL}} \Delta$.

With multiple-bank lending, we have to distinguish two cases, depending on whether $r^{\mathrm{ML}} D$ is above or below $\frac{R}{2}$.

Case (i): $r^{\mathrm{ML}} D < \frac{R}{2}$. Then, (C.3) becomes

$$\frac{\partial S^{\mathrm{ML}}}{\partial m^{\mathrm{ML}}} D = -\Delta r^{\mathrm{ML}} D \left(1 - p^{\mathrm{ML}}\right) \left(1 - m^{\mathrm{ML}}\right);$$

and (9) and (10) simplify respectively to

$$\Delta \left(1 - m^{\mathrm{ML}}\right) \left[\frac{R}{2} - r^{\mathrm{ML}} D \left(1 - p^{\mathrm{ML}}\right) \right] - c m^{\mathrm{ML}} = 0, \tag{C.5}$$

$$r^{\mathrm{ML}} p^{\mathrm{ML}} \left(2 - p^{\mathrm{ML}}\right) = y, \tag{C.6}$$

where $p^{\mathrm{ML}} = p_L + M^{\mathrm{ML}} \Delta = p_L + (2m^{\mathrm{ML}} - (m^{\mathrm{ML}})^2) \Delta$. We can then rearrange (C.6) and substitute it in (C.5) to get

$$\Delta \left(R - \frac{yD}{p^{\mathrm{ML}}} \right) = 2c \frac{m^{\mathrm{ML}}}{(1 - m^{\mathrm{ML}})} - \frac{\Delta y D}{2 - p^{\mathrm{ML}}}. \tag{C.7}$$

We can then compare M^{ML} and m^{IL} by using (C.4) and (C.7). It follows that $M^{\mathrm{ML}} > m^{\mathrm{IL}}$ if

$$2\frac{m^{\mathrm{ML}}}{(1-m^{\mathrm{ML}})} - \frac{\Delta y D}{c(2-p^{\mathrm{ML}})} > m^{\mathrm{IL}}.$$

Define the left-hand side of the above inequality as a generic function of $m \in [0,1]$ as

$$f(m) = 2\frac{m}{(1-m)} - \frac{\Delta y D}{c(2-p_L-(2m-m^2)\Delta)}.$$

The function gives the values $f(0) = -\frac{\Delta y D}{c(2-p_L)} < 0$ and $f(1) \to \infty$, and it is monotonically increasing in $m \in [0,1]$. Thus, there must exist a value $\overline{m} \in (0,1]$ such that $f(\overline{m}) = m^{\mathrm{IL}}$. This implies that $M^{\mathrm{ML}} < m^{\mathrm{IL}}$ if $m^{\mathrm{ML}} < \overline{m}$, and $M^{\mathrm{ML}} \geqslant m^{\mathrm{IL}}$ if $m^{\mathrm{ML}} \geqslant \overline{m}$ where $\overline{m} = f^{-1}(m^{\mathrm{IL}})$.

Case (ii): $r^{\mathrm{ML}}D > \frac{R}{2}$. In this case, (C.3) is equal to

$$\frac{\partial S^{\mathrm{ML}}}{\partial m^{\mathrm{ML}}}D = (1-m^{\mathrm{ML}})\Delta\left\{-r^{\mathrm{ML}}D(1-p^{\mathrm{ML}}) + \left(r^{\mathrm{ML}}D - \frac{R}{2}\right)(1-2p^{\mathrm{ML}})\right\};$$

(9) simplifies to

$$\Delta(1-m^{\mathrm{ML}})(R - r^{\mathrm{ML}}D)p^{\mathrm{ML}} - cm^{\mathrm{ML}} = 0, \tag{C.8}$$

and (10) to

$$(p^{\mathrm{ML}})^2 r^{\mathrm{ML}} + \frac{R}{D}p^{\mathrm{ML}}(1-p^{\mathrm{ML}}) = y. \tag{C.9}$$

We can then rewrite (C.9) as

$$(R - r^{\mathrm{ML}}D)p^{\mathrm{ML}} = R - \frac{yD}{p^{\mathrm{ML}}}$$

and substitute this in (C.8) to obtain

$$\Delta\left(R - \frac{yD}{p^{\mathrm{ML}}}\right) = c\frac{m^{\mathrm{ML}}}{(1-m^{\mathrm{ML}})}, \tag{C.10}$$

where $p^{\mathrm{ML}} = p_L + M^{\mathrm{ML}}\Delta$. We can then compare m^{IL} and M^{ML} using (C.4) and (C.10). We define the right-hand side of (C.10) for a generic $m \in [0,1]$ as

$$g(m) = \frac{m}{(1-m)}.$$

The function gives values $g(0) = 0$ and $g(1) \to \infty$, and it is monotonically increasing in $m \in [0,1]$. Thus, there must exist a threshold value $\overline{m} \in (0,1]$ such that $g(\overline{m}) = m^{\mathrm{IL}}$. This implies that $M^{\mathrm{ML}} < m^{\mathrm{IL}}$ if $m^{\mathrm{ML}} < \overline{m}$, and $M^{\mathrm{ML}} \geqslant m^{\mathrm{IL}}$ if $m^{\mathrm{ML}} \geqslant \overline{m}$ where $\overline{m} = g^{-1}(m^{\mathrm{IL}})$. □

Proof of Proposition 4. From the proof of Proposition 3, $\overline{m}$ is defined as $\overline{m} = f^{-1}(m^{\mathrm{IL}})$ in Case (i) and $\overline{m} = g^{-1}(m^{\mathrm{IL}})$ in Case (ii). Since both $f(m)$ and $g(m)$ are increasing monotonic functions, $\overline{m}$ is like this as well. It follows that $\overline{m}$ also increases with m^{IL}. From Eq. (C.4), it can be easily seen that m^{IL} increases with E, where $E = 1 - D$, and R, and decreases with c. The proposition follows. □

Proof of Corollary 1. With $E = 1$, the first-order condition for the monitoring efforts simplifies to

$$\Delta R - cm^{\mathrm{IL}} = 0,$$

with individual-bank lending, and

$$\Delta \frac{R}{2}\left(1-m^{\mathrm{ML}}\right)-cm^{\mathrm{ML}}=0,$$

with multiple-bank lending. Solving these equations, we obtain $m^{\mathrm{IL}}=\frac{\Delta R}{c}$ and $m^{\mathrm{ML}}=\frac{\frac{\Delta R}{c}}{\frac{\Delta R}{c}+2}$ and from here also $M^{\mathrm{ML}}=2m^{\mathrm{ML}}-(m^{\mathrm{ML}})^2=M^{\mathrm{ML}}=\frac{\frac{\Delta R}{c}(\frac{\Delta R}{c}+4)}{(\frac{\Delta R}{c}+2)^2}$. It is then easy to show that

$$m^{\mathrm{IL}}-M^{\mathrm{ML}}=\frac{(\frac{\Delta R}{c})^2(\frac{\Delta R}{c}+3)}{(\frac{\Delta R}{c}+2)^2}>0,$$

so that the corollary follows. □

Proof of Lemma 1. Recall that $p^{\mathrm{IL}}=p_L+m^{\mathrm{IL}}\Delta$ and $p^{\mathrm{ML}}=p_L+M^{\mathrm{ML}}\Delta$. Then the difference between (11) and (12) is given by

$$\pi^{\mathrm{IL}}-\pi^{\mathrm{ML}}=\left(m^{\mathrm{IL}}-M^{\mathrm{ML}}\right)\Delta R-c\left[\frac{(m^{\mathrm{IL}})^2}{2}-\left(m^{\mathrm{ML}}\right)^2\right],$$

and we can define the difference in costs as

$$\Gamma=c\left[\frac{(m^{\mathrm{IL}})^2}{2}-\left(m^{\mathrm{ML}}\right)^2\right].$$

Then we have $\Gamma>0$ if $m^{\mathrm{ML}}<\frac{m^{\mathrm{IL}}}{\sqrt{2}}$ and $\Gamma<0$ if $m^{\mathrm{ML}}>\frac{m^{\mathrm{IL}}}{\sqrt{2}}$. It follows that $M^{\mathrm{ML}}>m^{\mathrm{IL}}$ is a necessary condition for $\pi^{\mathrm{ML}}>\pi^{\mathrm{IL}}$ if $m^{\mathrm{ML}}>\frac{m^{\mathrm{IL}}}{\sqrt{2}}$, and it is a sufficient condition if $m^{\mathrm{ML}}<\frac{m^{\mathrm{IL}}}{\sqrt{2}}$. □

Proof of Proposition 6. For a given r, the bank chooses m_i to maximize (13) where

$$S=r(1-p_i)(1-p_{-i})+\frac{1}{D_2}\max\{rD_2-R,0\}\left[p_i(1-p_{-i})+p_{-i}(1-p_i)\right],$$

for $i\in\{1,2\}$ and $i\neq -i$. The first-order condition equals

$$\frac{\partial\pi}{\partial m_i}=\Delta R+\frac{\partial S}{\partial m_i}D_2-cm_i=0, \tag{C.11}$$

where

$$\frac{\partial S}{\partial m_i}=-r(1-p_{-i})\Delta+\frac{1}{D_2}\max\{rD_2-R,0\}(1-2p_{-i})\Delta.$$

In the symmetric case $p_i=p_{-i}=p^\ell, m_i=m_{-i}=m^\ell$, (C.11) becomes (14). Setting $[r-S]=y$ after substituting m^ℓ gives (15). □

Proof of Proposition 7. The proof is similar to that of Proposition 3. We compare m^ℓ and M^{ML} in equilibrium by substituting the investors' individual rationality constraints in the respective first-order conditions for the monitoring efforts. The only difference is that now also with individual-bank lending we have to distinguish two cases, as $r^\ell D_2$ can be above or below R. We then have to compare four cases, depending on whether $r^\ell D_2$ and $r^{\mathrm{ML}}D_1$ are above or below R and $\frac{R}{2}$, respectively. For the sake of brevity, we limit here the proof to two cases. The proof of the remaining cases is available from the authors upon request.

Case (i): $r^\ell D_2 < R$ and $r^{\mathrm{ML}} D_1 < \frac{R}{2}$. Substituting (15) in (14) with $D_2 = 2 - E$, we have

$$\Delta\left[R - (2-E)\frac{y(1-p^\ell)}{p^\ell(2-p^\ell)}\right] = cm^\ell, \tag{C.12}$$

where $p^\ell = p_L + m^\ell \Delta$. The case with multiple-bank lending is the same as in the proof of Proposition 3. We can then compare m^ℓ and M^{ML} by using (C.12) and (C.7), where $D_1 = 1 - E$. To do this, we rearrange (C.7) as

$$\Delta\left[R - (2-E)\frac{y(1-p^{\mathrm{ML}})}{p^{\mathrm{ML}}(2-p^{\mathrm{ML}})}\right] = c\frac{2m^{\mathrm{ML}}}{(1-m^{\mathrm{ML}})} - \frac{\Delta y E(1-p^{\mathrm{ML}})}{p^{\mathrm{ML}}(2-p^{\mathrm{ML}})},$$

where $p^{\mathrm{ML}} = p_L + M^{\mathrm{ML}}\Delta = p_L + (2m^{\mathrm{ML}} - (m^{\mathrm{ML}})^2)\Delta$. It follows that $M^{\mathrm{ML}} > m^\ell$ if

$$\frac{2m^{\mathrm{ML}}}{(1-m^{\mathrm{ML}})} - \frac{\Delta y E(1-p^{\mathrm{ML}})}{cp^{\mathrm{ML}}(2-p^{\mathrm{ML}})} > m^\ell.$$

Define the left-hand side of the above inequality as a generic function of $m \in [0, 1]$ as

$$h(m) = \frac{2m}{(1-m)} - \frac{\Delta y E[1 - p_L - (2m - m^2)\Delta]}{c[p_L + (2m-m^2)\Delta][2 - p_L - (2m-m^2)\Delta]}.$$

The function gives the values $h(0) = -\frac{\Delta y E(1-p_L)}{cp_L(2-p_L)} < 0$ and $h(1) \to \infty$, and it is monotonically increasing in $m \in [0, 1]$. Thus, there must exist a value $\widehat{m} \in (0, 1]$ such that $h(\widehat{m}) = m^\ell$. This implies that $M^{\mathrm{ML}} < m^\ell$ if $m^{\mathrm{ML}} < \widehat{m}$, and $M^{\mathrm{ML}} \geqslant m^\ell$ if $m^{\mathrm{ML}} \geqslant \widehat{m}$ where $\widehat{m} = h^{-1}(m^\ell)$.

Case (ii): $r^\ell D_2 > R$ and $r^{\mathrm{ML}} D_1 > \frac{R}{2}$. As before, we substitute (15) in (14) with $D_2 = 2 - E$ and obtain

$$\Delta\left[R - \left(\frac{2-E}{2}\right)\frac{y}{p^\ell}\right] = c\frac{m^\ell}{2},$$

where $p^\ell = p_L + m^\ell \Delta$. To compare m^ℓ and M^{ML}, we then rearrange (C.10) with $D_1 = 1 - E$ as

$$\Delta\left[R - \left(\frac{2-E}{2}\right)\frac{y}{p^{\mathrm{ML}}}\right] = c\frac{m^{\mathrm{ML}}}{(1-m^{\mathrm{ML}})} - \frac{\Delta y E}{2p^{\mathrm{ML}}}, \tag{C.13}$$

where $p^{\mathrm{ML}} = p_L + M^{\mathrm{ML}}\Delta = p_L + (2m^{\mathrm{ML}} - (m^{\mathrm{ML}})^2)\Delta$. Defining the right-hand side of (C.13) as a generic function of $m \in [0, 1]$ as

$$\varphi(m) = \frac{2m}{(1-m)} - \frac{\Delta y E}{c[p_L + (2m - m^2)\Delta]},$$

we have that $\varphi(0) = -\frac{\Delta y E}{cp_L} < 0$ and $\varphi(1) \to \infty$, with $\varphi(m)$ being monotonically increasing in $m \in [0, 1]$. Thus, there must exist a value $\widehat{m} \in (0, 1]$ such that $\varphi(\widehat{m}) = m^\ell$. This implies that $M^{\mathrm{ML}} < m^\ell$ if $m^{\mathrm{ML}} < \widehat{m}$, and $M^{\mathrm{ML}} \geqslant m^\ell$ if $m^{\mathrm{ML}} \geqslant \widehat{m}$ where $\widehat{m} = \varphi^{-1}(m^\ell)$.

For the same argument as in the proof of Proposition 4, the thresholds $\widehat{m} = h^{-1}(m^\ell)$ and $\widehat{m} = \varphi^{-1}(m^\ell)$ are increasing in m^ℓ, and are thus increasing in E. $\square$

Proof of Proposition 8. For a given r_j, each bank j chooses m_{ij} to maximize (B.2). The first-order condition is given by

$$\frac{\partial \pi_j}{\partial m_{ij}} = (1-m_{i,-j})^{k-1}\Delta \sum_{v=0}^{N} \max\left\{v\frac{R}{k} - r_j D, 0\right\}\left[\binom{N-1}{v-1}(1-p_{-i})^{N-v} p_{-i}^{v-1} - \binom{N-1}{v} p_{-i}^{v}(1-p_{-i})^{N-v-1}\right] - cm_{ij} = 0$$

for $i \in \{1,\ldots,N\}$, $i \neq -i$, $j \in \{1,\ldots,k\}$ and $j \neq -j$, when all $-j$ banks exert an effort $m_{i,-j}$ on project i and all other $-i$ projects have a success probability p_{-i}. The second-order condition is simply

$$\frac{\partial^2 \pi_j}{\partial m_{ij}^2} = -c < 0.$$

The negative sign of the second-order condition for any $m_{i,-j}$ indicates again that banks' expected profits are globally concave and consequently that the first-order conditions are binding in equilibrium. Furthermore, from the ratio of the first-order conditions for project i, it is easy to derive

$$\frac{\partial \pi_j}{\partial m_{ij}} \Big/ \frac{\partial \pi_{-j}}{\partial m_{i,-j}} = 0.$$

It follows that the equilibrium in the monitoring efforts is unique and symmetric. Using then $m_{ij} = m(k)$, $p_i = p_{-i} = p(k)$ and $r_j = r_{-j} = r(k)$, the expression of the shortfalls in (B.3) simplifies to

$$S(k) = \frac{1}{D}\sum_{v=0}^{N}\binom{N}{v}\max\left\{r(k)D - v\frac{R}{k}, 0\right\} p^v(k)\big(1-p(k)\big)^{N-v},$$

where v is the number of successful projects. Then we can rearrange the first-order condition as in (18), where

$$\frac{\partial S(k)}{\partial m(k)}D = \frac{1}{N}\sum_{v=0}^{N}\binom{N}{v}\max\left\{r(k)D - v\frac{R}{k}, 0\right\} p^{v-1}(k)\big(1-p(k)\big)^{N-v-1} \times\big[v - Np(k)\big]\big(1-m(k)\big)^{k-1}\Delta. \tag{C.14}$$

Finally, setting $[r - S] = y$ after substituting $m(k)$ implies (19). □

Proof of Lemma 2. We have to show that $\lim_{k\to\infty} |\frac{\partial S(k)}{\partial m(k)}D| = 0$. To do this we first find a quantity $\Theta(k)$ greater than $|\frac{\partial S(k)}{\partial m(k)}D|$ for any k, where $\frac{\partial S(k)}{\partial m(k)}D$ is given by (C.14). Then we show that $\lim_{k\to\infty}\Theta(k) = 0$ so that also $\lim_{k\to\infty} |\frac{\partial S(k)}{\partial m(k)}D| = 0$ holds. Take $\Theta(k)$ as being equal to

$$\Theta(k) = \frac{1}{N}\sum_{v=0}^{N}\binom{N}{v} r(k)Dp^{v-1}(k)\big(1-p(k)\big)^{N-v-1}[Np_H - v]\Delta.$$

This is greater than $|\frac{\partial S(k)}{\partial m(k)}D|$ because $r(k)D \geqslant \max\{r(k)D - v(\frac{R}{k}), 0\}$, $1 \geqslant (1-m(k))^{k-1}$, and $Np_H \geqslant Np(k)$. The expression for $\Theta(k)$ becomes

$$\Theta(k) = \frac{\Delta r(k)D}{p(k)(1-p(k))}\left\{p_H \sum_{v=0}^{N}\binom{N}{v} p^v(k)\bigl(1-p(k)\bigr)^{N-v} - \frac{1}{N}\sum_{v=0}^{N}\binom{N}{v} v p^v(k)\bigl(1-p(k)\bigr)^{N-v}\right\}$$
$$= \frac{\Delta r(k)D}{p(k)(1-p(k))}\bigl[p_H - p(k)\bigr]$$

since, from the binomial distribution we have that $\sum_{v=0}^{N}\binom{N}{v}p^v(k)(1-p(k))^{N-v} = 1$ and $\sum_{v=0}^{N}\binom{N}{v}vp^v(k)(1-p(k))^{N-v} = Np(k)$. As $k \to \infty$, $p(k) = p_H$, because $(1-m(k))^k \to 0$. It follows that $\lim_{k\to\infty}|D\frac{\partial S(k)}{\partial m(k)}| = 0$ as $\lim_{k\to\infty}\Theta(k) = 0$. □

Proof of Proposition 9. The proof follows immediately from Lemma 2 and from the fact that the left-hand side of (18) becomes negative as $k \to \infty$. □

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